

# Self-Loop Aggregation Product — A New Hybrid Approach to On-the-Fly LTL Model Checking

Alexandre Duret-Lutz (LRDE/EPITA)

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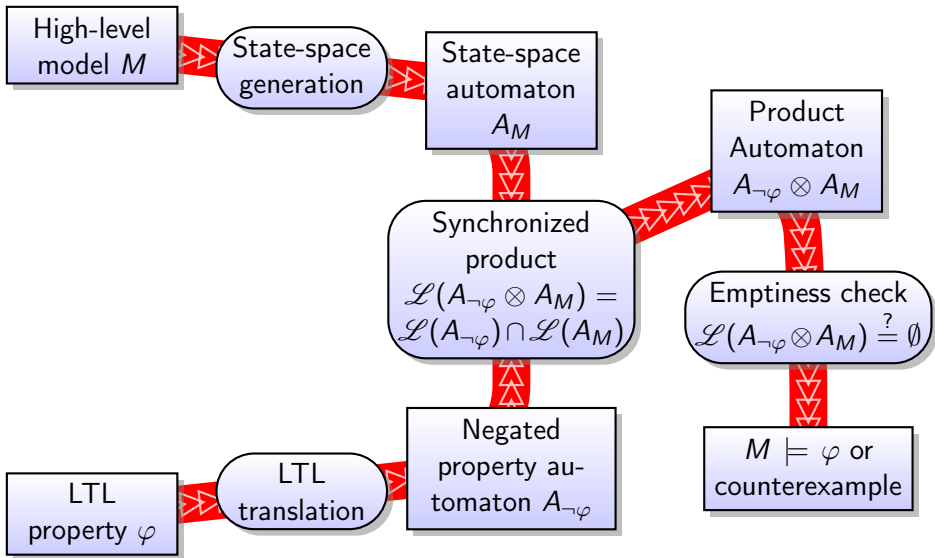
Yann Thierry-Mieg (LIP6/Paris 6)

ATVA 2011

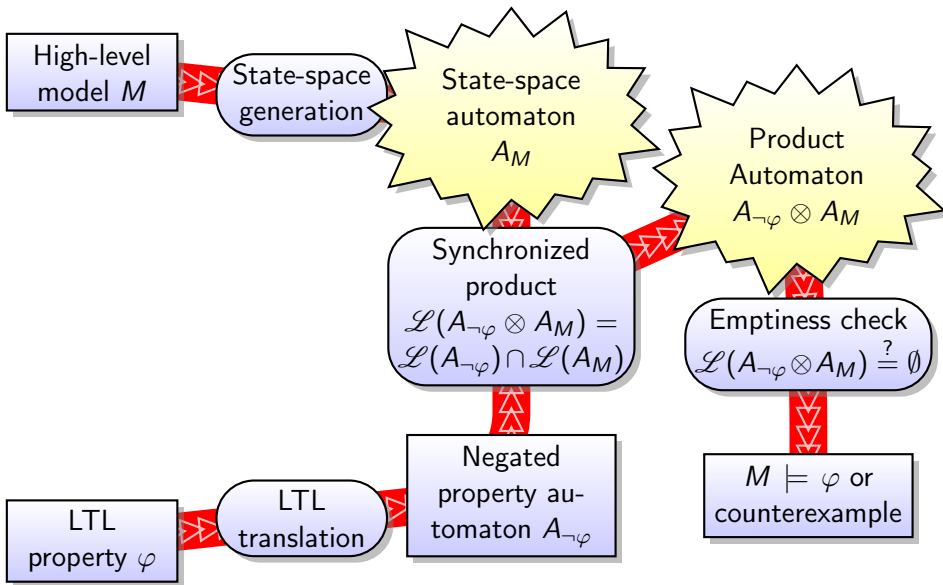
October 2011

[http://move.lip6.fr/software/DDD/ltl\\_bench.html](http://move.lip6.fr/software/DDD/ltl_bench.html)

# Automata-Theoretic Explicit LTL Model Checking



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High-level  
model  $M$

On-the-fly generation  
of state-space automaton  
 $A_M$

Product  
Automaton  
 $A_{\neg\varphi} \otimes A_M$

Synchronized  
product  
 $\mathcal{L}(A_{\neg\varphi} \otimes A_M) =$   
 $\mathcal{L}(A_{\neg\varphi}) \cap \mathcal{L}(A_M)$

Emptiness check  
 $\mathcal{L}(A_{\neg\varphi} \otimes A_M) \stackrel{?}{=} \emptyset$

LTL  
property  $\varphi$

LTL  
translation

Negated  
property au-  
tomaton  $A_{\neg\varphi}$

$M \models \varphi$  or  
counterexample

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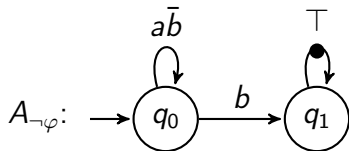
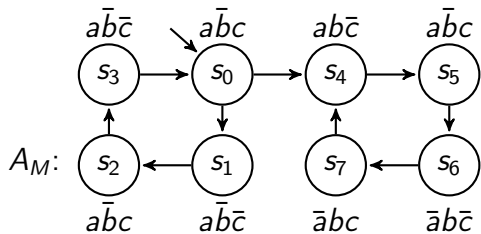
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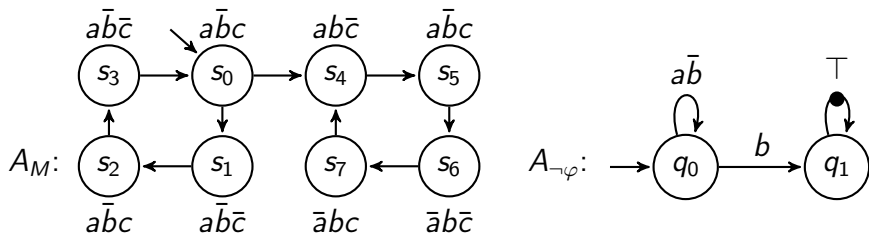
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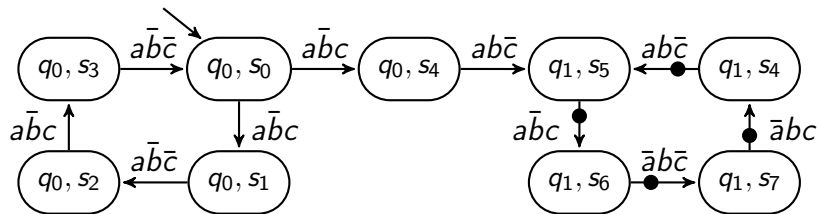
# Explicit Approach



# Explicit Approach

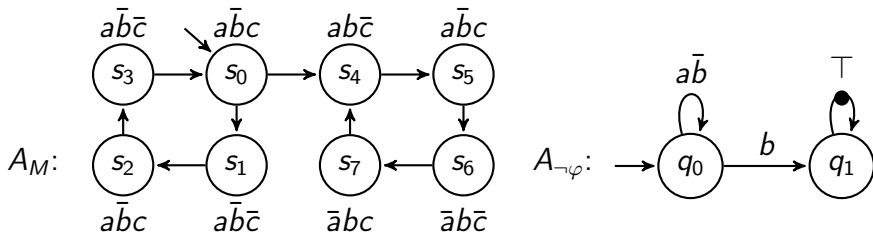


$$A_{\neg\varphi} \otimes A_M:$$



- Emptiness check = search for an accepting cycle in the product
- State explosion problem

# Symbolic Observation Graph (SOG)



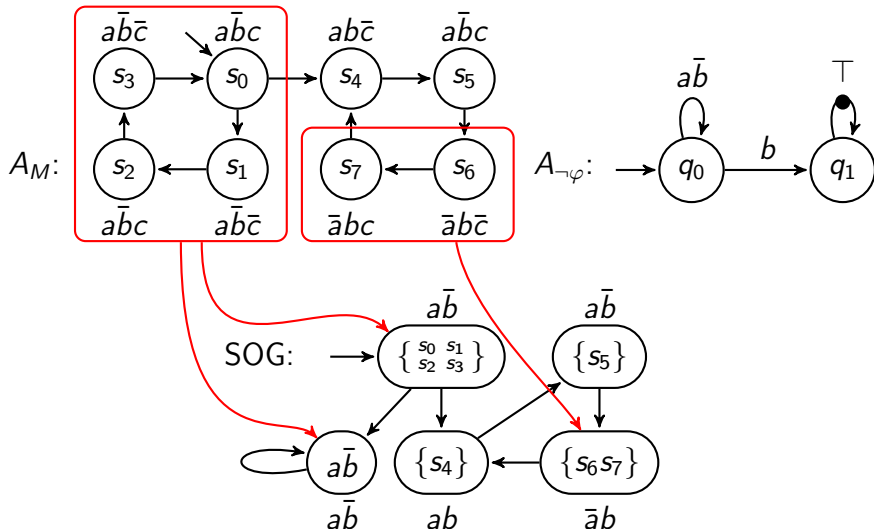
- For stuttering invariant properties (e.g.,  $LTL \setminus \mathbf{X}$ )
- Ignore non-observable propositions
- Aggregate Kripke states with homogeneous labels
- Represent aggregates using BDDs



K. Klai and D. Poitrenaud. MC-SOG: An LTL model checker based on symbolic observation graphs. In Proc. of PN'08, vol. 5062 of LNCS, pp. 288–306. Springer



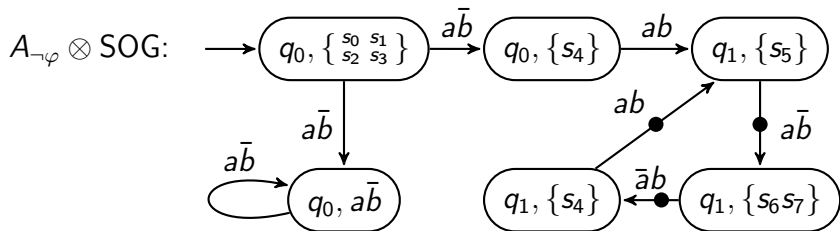
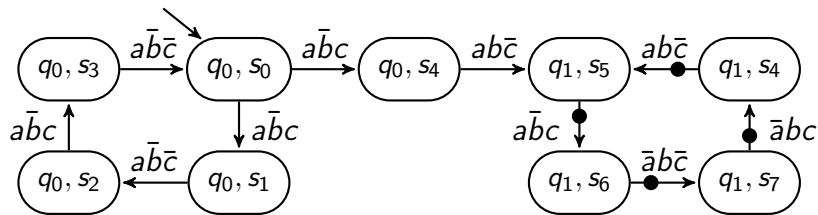
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K. Klai and D. Poitrenaud. MC-SOG: An LTL model checker based on symbolic observation graphs. In Proc. of PN'08, vol. 5062 of LNCS, pp. 288–306. Springer

# Product Sizes: Kripke vs. SOG

$$A_{\neg\varphi} \otimes A_M:$$



K. Klai and D. Poitrenaud. MC-SOG: An LTL model checker based on symbolic observation graphs. In Proc. of PN'08, vol. 5062 of LNCS, pp. 288–306. Springer

# BCZ: Multiple-State Tableaux

- Similar to SOG with only 1 step par aggregate:
  - supports full LTL
  - no need to search for livelock cycles
- Low aggregation power: on our example with low branching, it does not reduce the Kripke structure.



A. Biere, E. M. Clarke, and Y. Zhu. Multiple state and single state tableaux for combining local and global model checking. In *Correct System Design*, vol. 1710 of LNCS, pp. 163–179. Springer, 1999

# Building a Product Directly

High-level  
model  $M$

On-the-fly generation  
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 $A_M$

On-the-fly  
synchronized product  
 $\mathcal{L}(A_{\neg\varphi} \otimes A_M) =$   
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Emptiness check  
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LTL  
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LTL  
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Negated  
property au-  
tomaton  $A_{\neg\varphi}$

$M \models \varphi$  or  
counterexample

# Building a Product Directly

High-level  
model  $M$

Dynamic and on-the-fly generation  
of an automaton  $D$  such that  
 $\mathcal{L}(D) = \emptyset \iff \mathcal{L}(A_{\neg\varphi} \otimes A_M) = \emptyset.$

Emptiness check  
 $\mathcal{L}(D) \stackrel{?}{=} \emptyset$

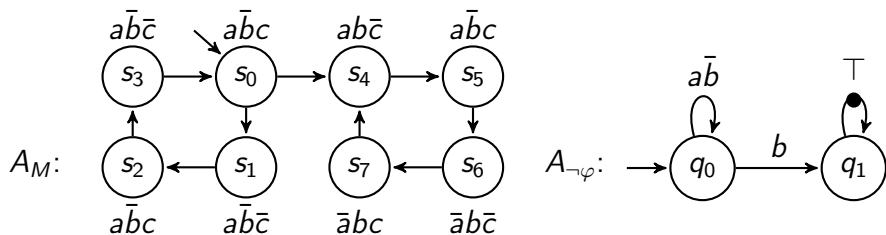
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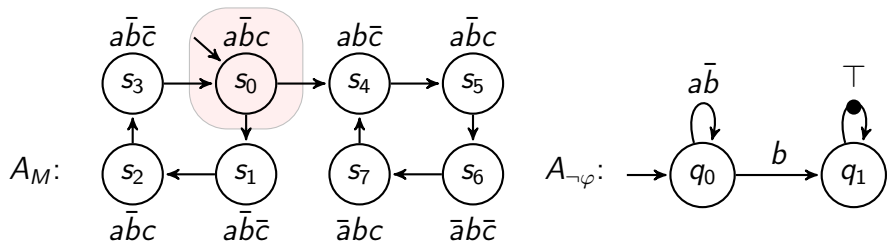
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# Self-Loop Aggregation Product (SLAP)



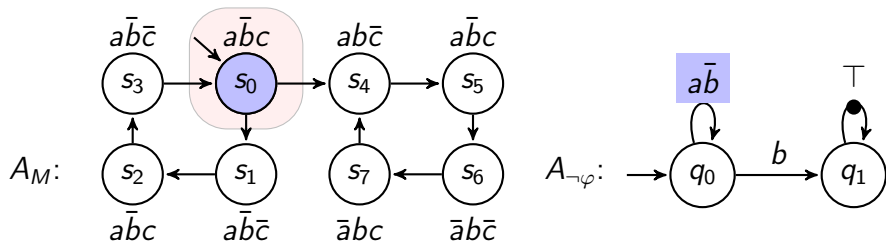
Self-Loop Aggregation **Product**:

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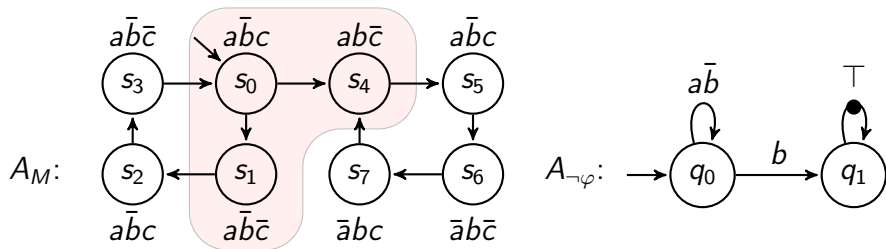
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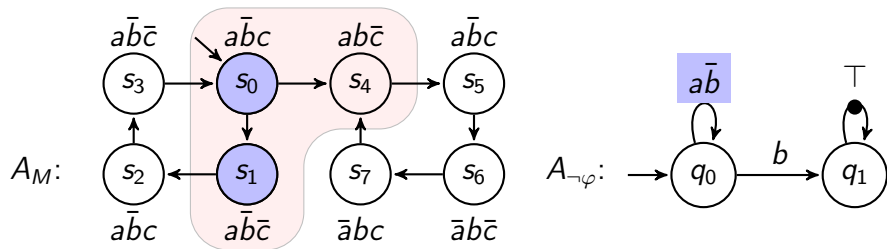


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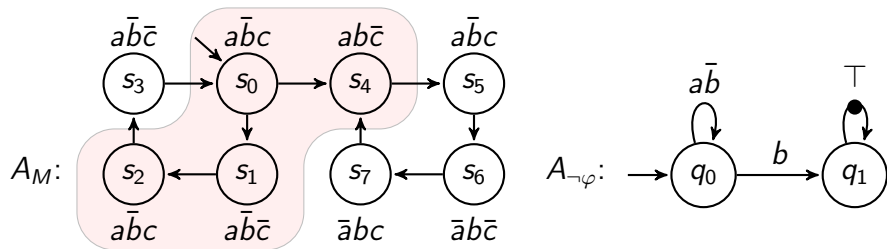
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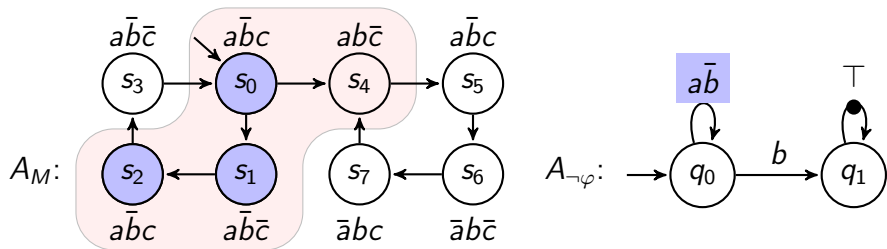
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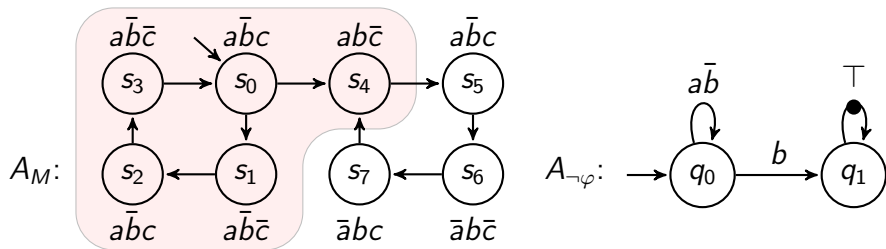
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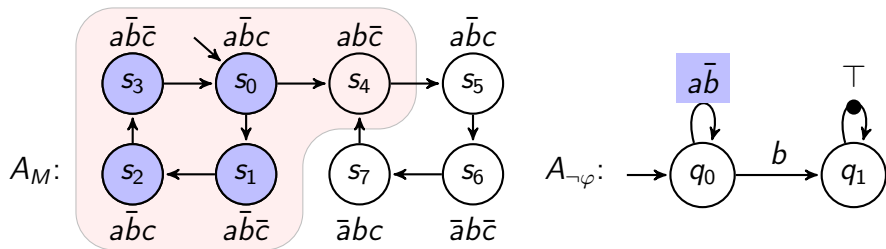
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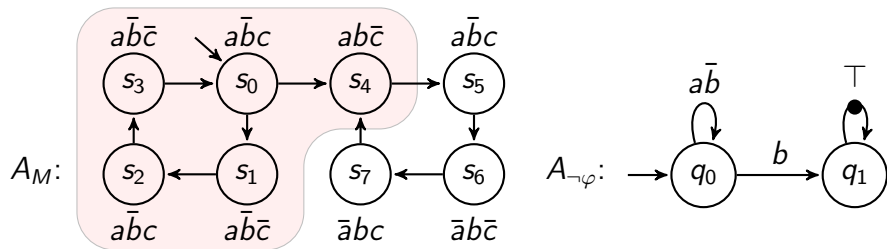
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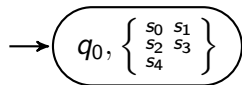


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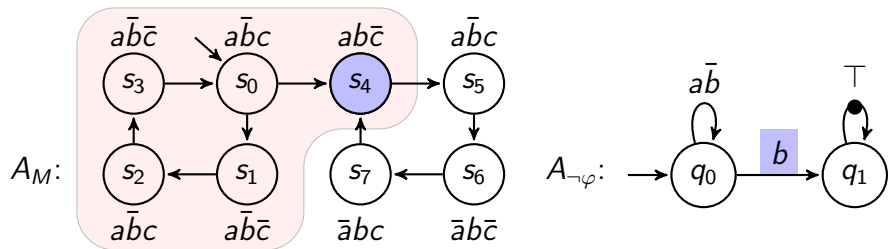
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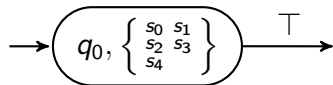
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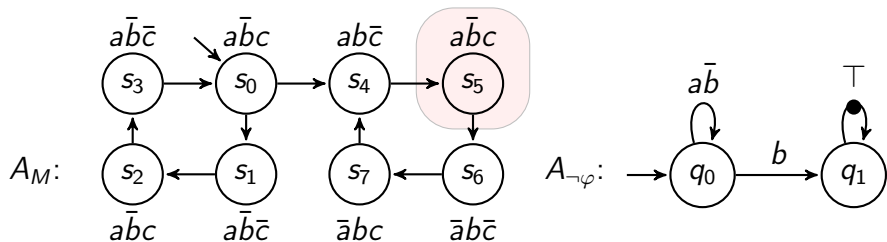


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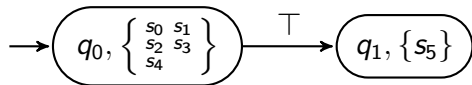




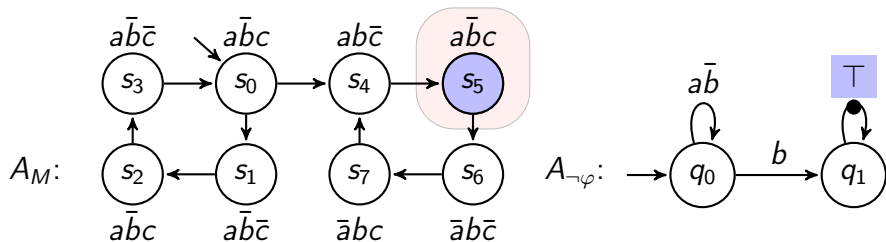
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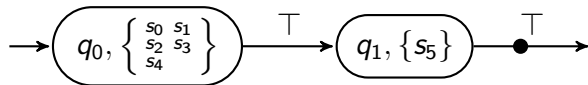
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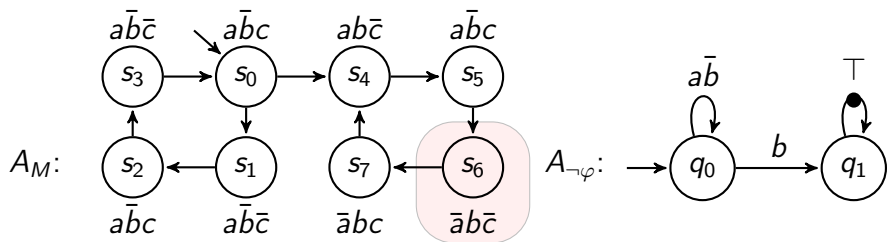
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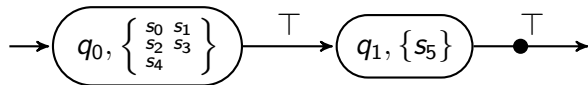
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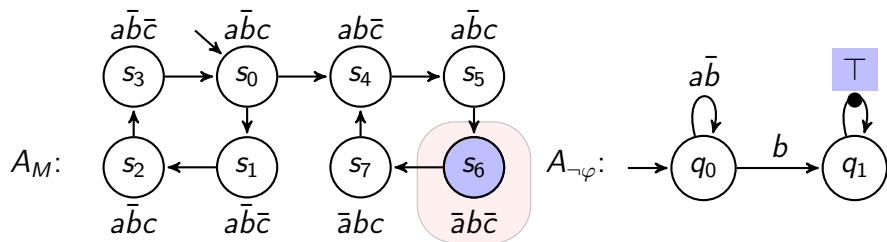
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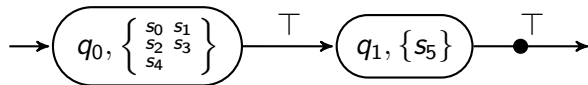
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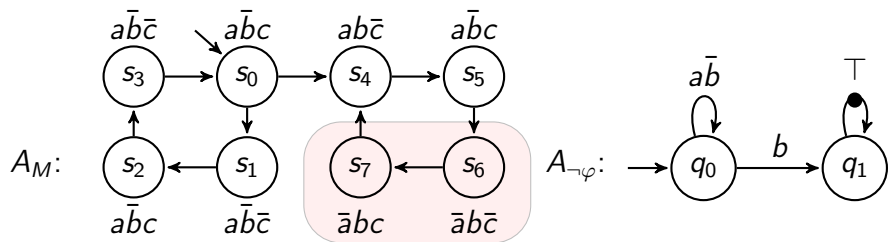
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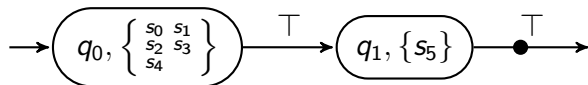
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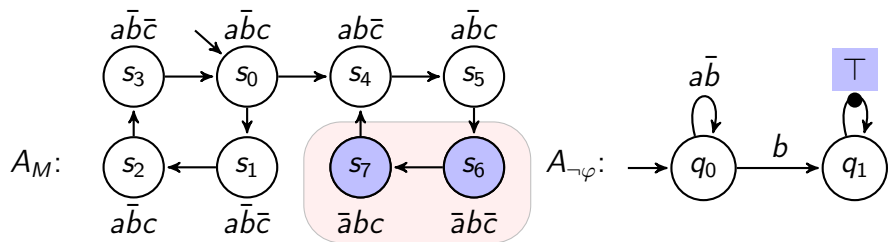
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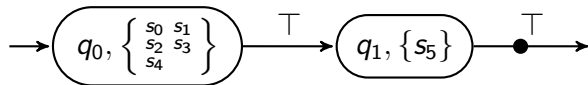
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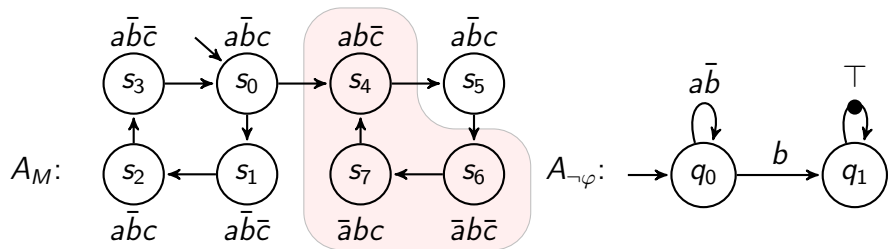
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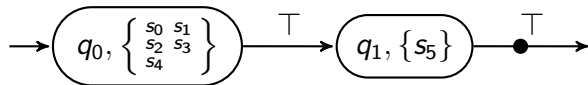
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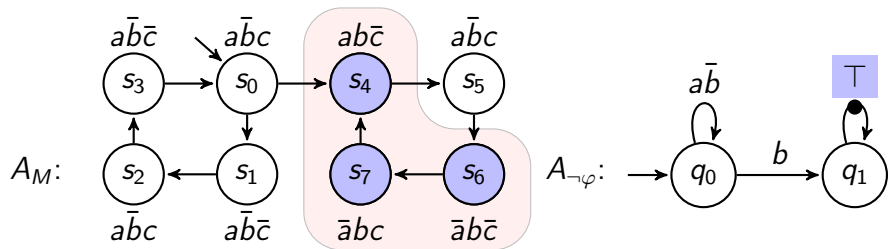
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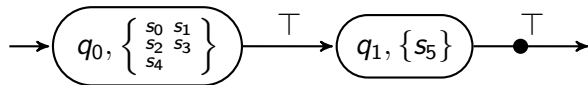
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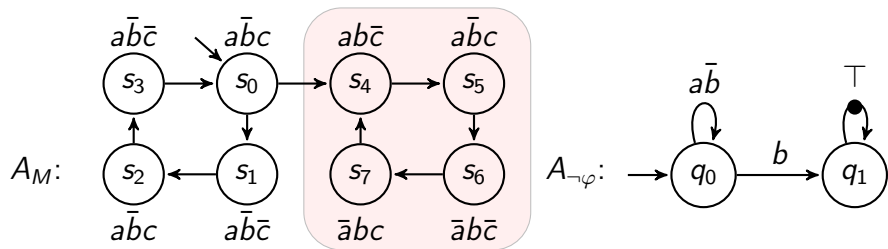


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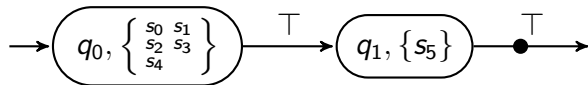




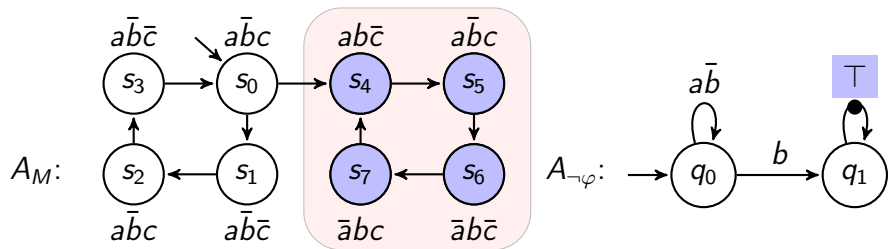
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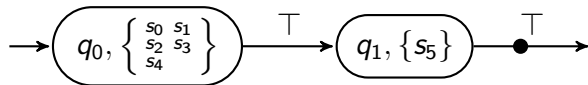
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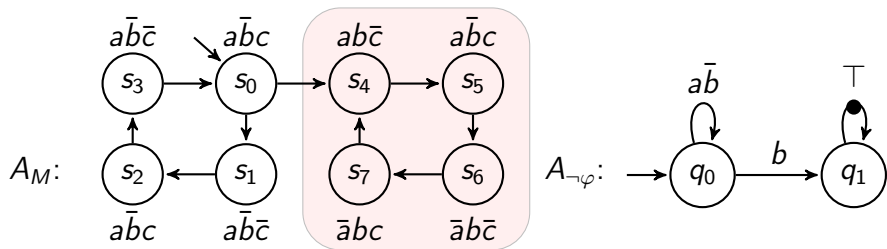
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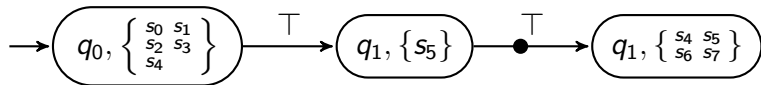
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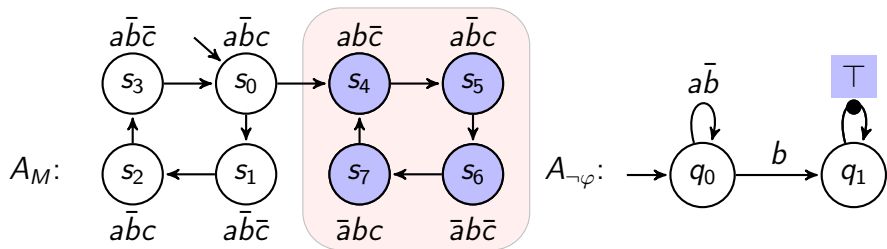
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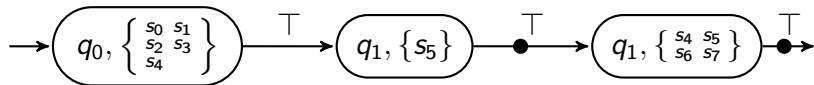
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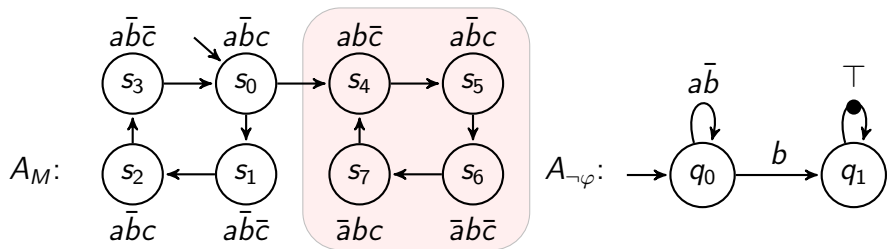
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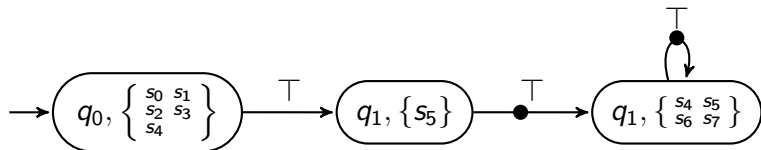
Self-Loop Aggregation Product:



# Self-Loop Aggregation Product (SLAP)



Self-Loop Aggregation Product:



# Fully Symbolic Approach (FS)

- Encode the Kripke structure and the property automaton using BDDs.
- Combine the two BDDs. (Symbolic product.)
- Use fixpoint computations to decide whether the product contains an accepting cycle.
- Two symbolic emptiness checks compared:
  - EL
  - OWCTY



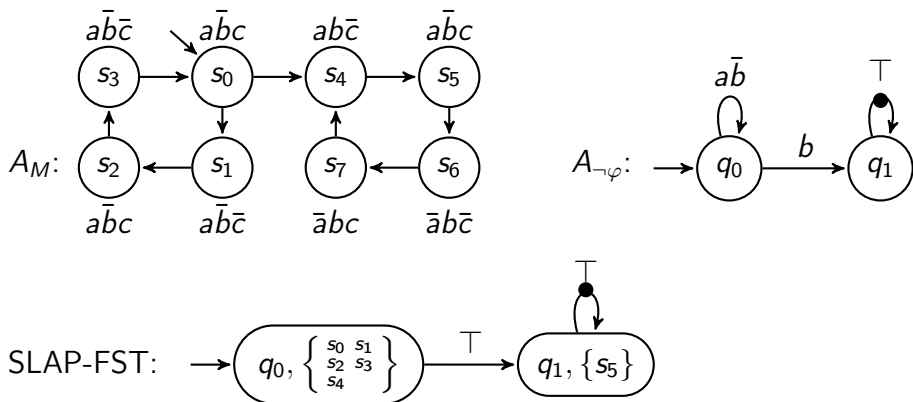
K. Fislser, R. Fraer, G. Kamhi, M. Y. Vardi, and Z. Yang. Is there a best symbolic cycle-detection algorithm? In Proc. of TACAS'01, vol. 2031 of LNCS, pp. 420–434. Springer



F. Somenzi, K. Ravi, and R. Bloem. Analysis of symbolic SCC hull algorithms. In Proc. of FMCAD'02, vol. 2517 of LNCS, pp. 88–105. Springer

# SLAP-FST

- FST = Fully Symbolic search in Terminal automata states
- When reaching a state of  $A_{\neg\varphi}$  which is terminal (and accepting) use a fully symbolic search.



# Summary of methods

	Explicit	BCZ	SOG	SLAP	PDP	FS
logic	LTL		LTL \ X		LTL	
prop. aut.	explicit					symb./expl.
data str.	graph	BDD+graph			BDD[]	BDD
empt. chk	explicit				symbolic (OWCTY/EL)	

PDP = Property Driven Partitioning,



R. Sebastiani, S. Tonetta, and M. Y. Vardi. Symbolic systems, explicit properties: on hybrid approaches for LTL symbolic model checking. In Proc. of CAV'05, vol. 3576 of LNCS, pp. 350–363. Springer



# Experimental Framework

## Implemented algorithms

BCZ, SOG, SLAP, SLAP-FST, EL, OWCTY

## Tools used

**SDD** Hierarchical Set Decision Diagrams (<http://ddd.lip6.fr/>)

- Hierarchy (memory gain)
- Automatic saturation (improves fixpoint computations)

**Spot** Model checking library (<http://spot.lip6.fr/>)

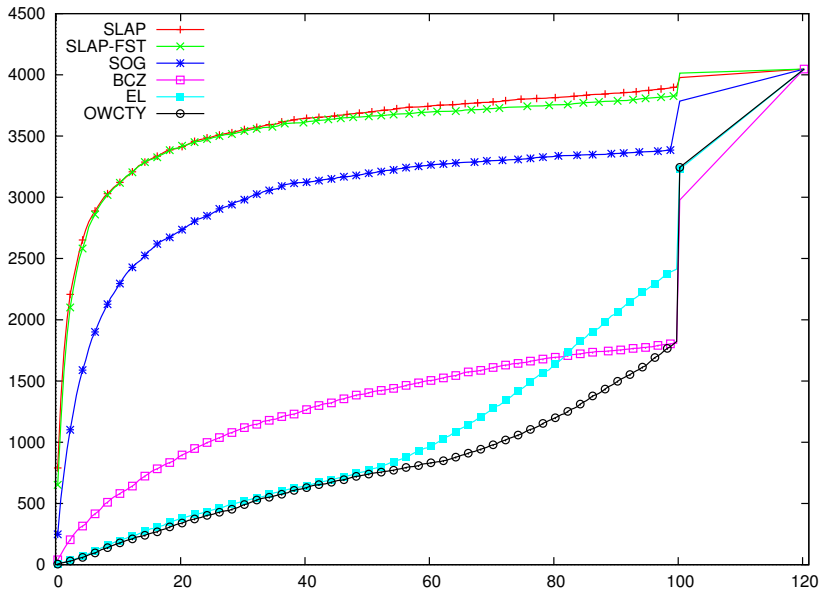
- Good LTL-to-TGBA translation
- Explicit emptiness-check algorithms

## Data

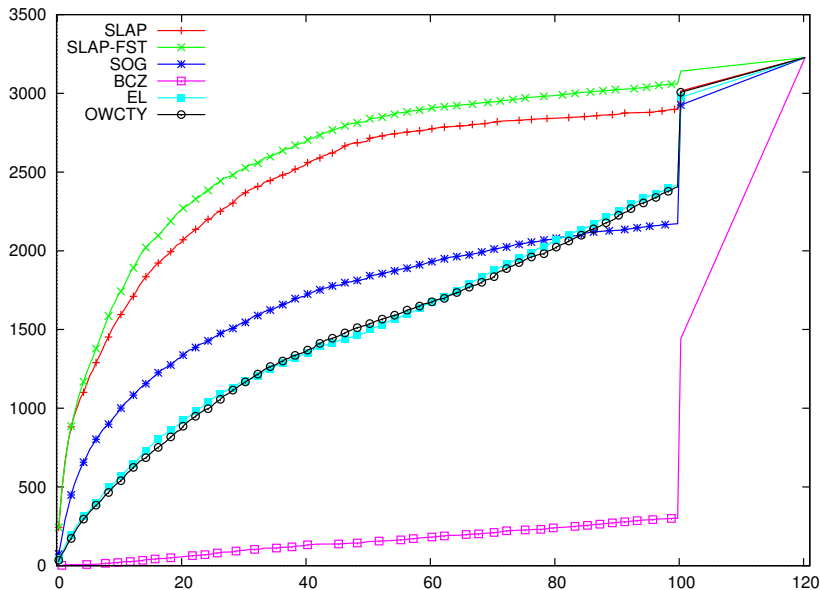
**Models** Scalable toy models with  $10^6 \dots 10^{66}$  states.

**Formulas** Random formulas (filtered), plus a set of (random) weak fairness properties.

# Cumulative Plot: Violated Formulas

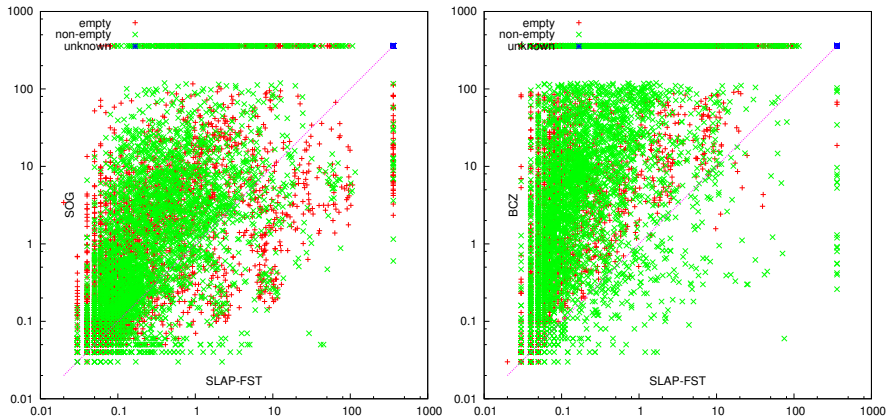


# Cumulative Plot: Verified Formulas



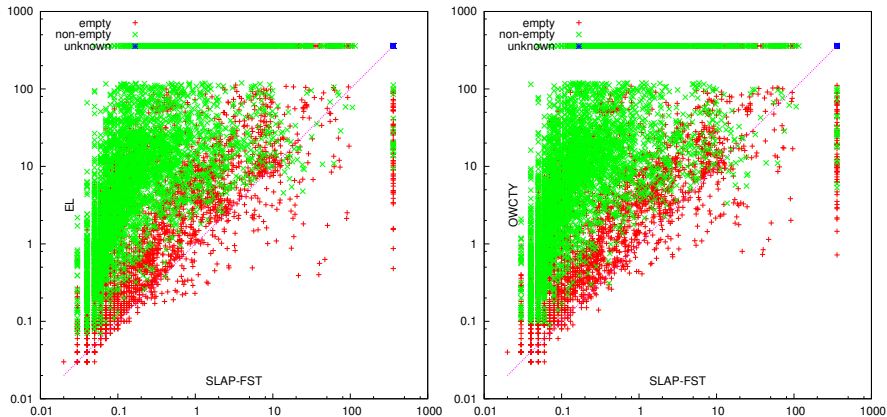
# Scatter Plots: SLAP-FST vs. Hybrid

Runtime in seconds. Timeout at 120s.



# Scatter Plots: SLAP-FST vs. Fully Symbolic

Runtime in seconds. Timeout at 120s.



# Conclusion

- SLAP, a new hybrid approach:
  - Aggregates states at the product level, not in the Kripke structure
  - Can therefore adjust dynamically to the features (self-loops and acceptance conditions) present in the property automaton.
  - SLAP-FST: a variant using a fully-symbolic algorithm for terminal states.
- SLAP-FST most competitive algorithm of those we compared, **on this benchmark**
- On-the-fly computations have strong impact when counterexamples exist. Hybrid approaches outperform fully symbolic approaches in these cases.

[http://move.lip6.fr/software/DDD/ltl\\_bench.html](http://move.lip6.fr/software/DDD/ltl_bench.html)